Lecture 5

Vector analysis — التحليل الاتجاهي:-

$$\iiint_V (\bar{\nabla} \cdot \bar{F})\, dv = \oiint_S \bar{F} \cdot d\bar{s}$$

$$\iint_S (\bar{\nabla} \times \bar{F}) \cdot d\bar{s} = \oint_C \bar{F} \cdot d\bar{r}$$

If $\vec{H} = \bar{\nabla} \times \bar{A}$ prove: $\oiint_S \vec{H} \cdot ds = 0$

sol.

$$\oiint_S \underbrace{\bar{F}}_{H} \cdot d\bar{s} = \iiint_V (\underbrace{\bar{\nabla} \cdot \bar{F}}_{\bar{\nabla} \cdot \bar{\nabla} \times A})\, dv = \iiint_V (\bar{\nabla} \cdot \bar{\nabla} \times \bar{A})\, dv$$

$$\oint_C \vec{E} \cdot d\vec{r} = \frac{-1}{c} \cdot \frac{\partial}{\partial t} \iint_S \vec{H} \cdot d\bar{s}$$

show $\bar{\nabla} \times \vec{E} = \frac{-1}{c} \frac{\partial \vec{H}}{\partial t}$

$$\oint_C \vec{E} \cdot d\vec{r} = \iint_S \bar{\nabla} \times E \cdot ds$$

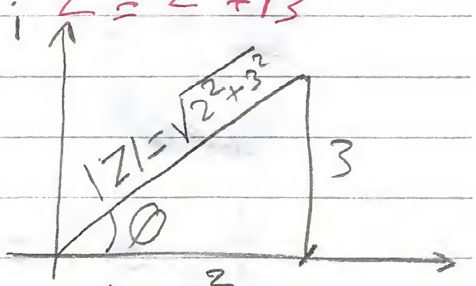
Complex analysis :- التحليل المركب

complex number :-

$1 = \sqrt{1}$ $\qquad\qquad$ $i = \sqrt{-1}$

Z

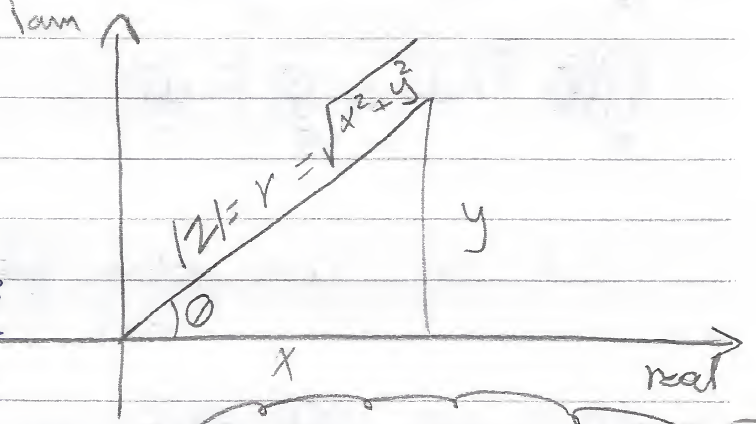
$Z = 2 + i3$



$\tan \phi = \frac{3}{2}$

Complex variable تغير المركب

$$Z = x + iy$$



$$\boxed{Z = r\cos\theta + i r\sin\theta}$$
$$= x + iy$$

$$e^{ix} = 1 + \frac{ix}{1!} + \frac{-x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!}$$

note: $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\sin x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$e^{ix} = \cos x + i\sin x$$

$$Z = r\cos\theta + ir\sin\theta = re^{i\theta}$$

$$\frac{dy}{dx} = y \qquad \frac{dy}{y} = dx$$

$$y = e^x$$

$$\frac{d}{dx} e^{ix} = e^{ix}$$

$$\frac{d}{dx} e^{ix} = ie^{ix}$$

$$\frac{d}{dx}(\cos x + i\sin x)$$

$$= -\sin x + i\cos x$$

$$= i\left(+\cos x - \frac{1}{i}\sin x\right)$$

$$= i(\cos x + i\sin x)$$

$$\therefore e^{ix} = \cos x + i\sin x$$

if $z = x + iy$

$$f(z) = u(x,y) + i\,v(x,y) = u + iv$$

if $z \to z_0 \quad \therefore x \to a_0, \; y \to b_0$

$f \longrightarrow f_0, \quad u \longrightarrow u_0, \quad v \longrightarrow v_0$

$(\Delta z = \Delta x + i\Delta y)$ وطبعا

ex.

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{u(z+\Delta z) + i\,v(z+\Delta z) - [u(z) + i\,v(z)]}{\Delta z}$$

if $\Delta z = \Delta x$

$$= \lim_{\Delta x \to 0} \frac{u(x+\Delta x) + i\,v(x+\Delta x) - [u(x) + i\,v(x)]}{\Delta x}$$

$$= \boxed{\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}} \longrightarrow (1)$$

if $\Delta z = i\Delta y$

$$f'(z) = \lim \frac{u(x, y+\Delta y) + i\,v(x, y+\Delta y) - [u(x,y) + i\,v(x,y)]}{i\Delta y}$$

$$= \frac{1}{i}\frac{\partial u}{\partial y} + \frac{i}{i}\frac{\partial v}{\partial y} = \boxed{-i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}} \longrightarrow (2)$$

لكي تكون الدالة $f(z)$ تحليلية:

$$\therefore \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

Cauchy - Rieman equation